MATHEMATICAL ASSOCIATION

supporting mathematics in education

Sixth-Form Mathematics Today

a short guide for lecturers in higher education

What qualifications will they have?

Students entering universities to read mathematics-based courses come from a wide variety of social and academic backgrounds, even within England. Differences in Scotland, Wales and Northern Ireland are discussed later.

The vast majority will come from a **Mathematics GCE** background, in which case they will be one of the roughly half their age cohort taking GCEs, of whom 50 000, about a fifth, take the full Mathematics A Level. (At present, about a third of students who embark on AS Mathematics do not complete the A Level; it is hoped that this situation will change with the introduction of modified specifications for first A-Level examining in June 2006 – see below.)

a wide variety of backgrounds and experiences

When they apply to university, the only concrete evidence of students' mathematical attainment may be their GCSE grades, which are of limited value. It is possible to gain an A grade at GCSE without getting any of the marks allocated for algebra; on the other hand, a student who has attended a school which does not offer the opportunity to do Higher level GCSE can gain at most a B, and may be potentially very capable. Some post-16 institutions insist on at least an A grade at GCSE before accepting a student on an A-Level course; others have an 'open access' policy, in which case teachers may have very mixed ability classes, which may limit their opportunities to target expectations appropriately. Institutions also vary enormously in the degree of independence students are expected to have with respect to their A-Level studies, so that some come to university very experienced in organising their own time to meet medium-term deadlines, or indeed to meet self-imposed deadlines, whereas others will have been lovingly nurtured so as to maximise their A-Level grades, but will have little experience of organising themselves to do so.

A Levels 2002 to 2005

A student applying to university who is predicted an **A grade** will be expected to be fluent in most aspects of the specification. Three of the six units contain the core of pure mathematical content which is common across the examining boards and accounts for nearly all the pure mathematics in A Level 'Mathematics'. The applied mathematics content varies enormously between options for a given board, and between boards (AQA, Edexcel and OCR), with mechanics modules being perceived to be the most demanding conceptually. A **C-grade** candidate will have mastered many of the important concepts encountered, although there may be significant holes in important areas, including algebraic manipulation, whereas an **E-grade** candidate will have struggled with many of the ideas met, typically being quite weak algebraically, and is likely to be expected to gain a high proportion of marks scored from the less conceptually demanding modules. For all students, those ideas which are typically encountered late in the course, such as integration by parts, are likely to be less well established than more basic ideas: the core content is available in the specifications of any of the examining boards, and will be seen to exclude for example complex numbers, hyperbolic functions and any but first order variables separable differential equations.

A Levels from 2006

For teaching from September 2004, so for first award in June 2006, the English and Welsh examining boards will be offering specifications modified in the light of the difficulties arising from Curriculum 2000. The present core pure mathematics content will be spread over four modules, C1 to C4, with candidates for a Mathematics A Level studying in addition two applied units. The objects of the changes are to enhance recruitment and retention to the course, and to boost mastery of the core content.

Further Mathematics

Many institutions are able to offer their most committed mathematicians at least an AS in Further Mathematics, and possibly an A Level. There exist a variety of projects to make these available by distance learning, and The Mathematical Association, among others, is working towards ensuring the availability of Further Mathematics to all students who would benefit from it. An A-Level student might take units varying from Pure 1–6 and Mechanics 1–6 to Pure 1–5, with Statistics 1–3, Discrete Maths 1–2 and Mechanics 1–2; these latter students would typically have a broader mathematical background, but much less experience of harder concepts and problems. About 5000 students annually take Further Mathematics A Level; they have spent a relatively large part of their post-16 study in Mathematics, and an A-grade Further Mathematics student would be very well-prepared for nearly all university mathematics courses.

Advanced Extension Award and STEP

Able students may be entered for the **Advanced Extension Award** in Mathematics, a demanding paper based on the A-Level core content and aimed at the top 25% of A grade candidates; it is has two grades (Distinction and Merit). Also, they may sit **STEP** papers 1, 2 or 3 (2 and 3 being the more demanding, with 3 requiring material beyond A Level). These are used for selection by Cambridge and some other universities but are available to anyone. Students receive very variable support for sitting these papers, with many schools feeling unable to provide the necessary expertise and/or time, but there is now online help available.

FSMQs and AS Use of Mathematics

Some students may come to university with Level 3 **Free Standing Mathematics Qualifications**; portfolios of evidence contributing half the marks in each unit. The content is comparable in standard to that of Mathematics AS units. Two of these units can be combined with a unit in Applying Mathematics to give AS **Use of Mathematics**; there is some choice of units, so students will not necessarily be familiar with calculus.

International Baccalaureate

About forty British institutions among over a thousand worldwide offer the International Baccalaureate Diploma course. It is a two-year programme designed for highly-motivated secondary students, who study three subjects at Higher Level (broadly comparable to A Level) and three at Standard Level (roughly AS level), as well as the theory of knowledge; they produce an extended essay, and also satisfy requirements in a Creativity/Action/Service section. The maximum score is 45 points with 7 points available in each of the six subjects, and a further 3 points in the other elements; the standard for 7 points is very high. Mathematics and Further Mathematics are available at Higher Level, and Mathematical Studies and Mathematical Methods at Standard Level; all students study a mathematical subject.

other qualifications

Increasingly, students will apply to university with a mixture of academic (as above) and vocational qualifications: **BTeC, HNC, HND, VCE** etc. The latter is the name for A-Level equivalent vocational courses, consisting of 6 units for the single awards and 12 for double awards, as A Levels do. Assessment is less through formal external written assessment and more by way of coursework, and students often have well-honed organisational and presentational skills as a result. Content for BTeC, HNC and HND courses varies widely by provider, even where titles are similar, but is often individualised, and students frequently emerge with good motivation and generic skills.

Where will they have studied?

colleges

Over half of English post-16 students are now in post-16 institutions, either **Sixth Form Colleges** (16–19 students) or **Colleges of Further Education** (a wider clientele). Either may exist alongside 11-18 schools, and sizes of these institutions are very variable. Both are likely to offer a variety of vocational courses as well as academic ones, and courses at a variety of levels. Colleges of FE in particular will boast students of a wide age-span, sometimes taking courses between employment, or on day-release or as evening courses, although most full-time students are likely to be in the 16-19 age group. All courses are likely to have a wide range of entry qualifications, and where numbers are small this can lead to very mixed ability classes for A Level Mathematics, for example. Larger colleges, though, can offer a big variety of courses, with perhaps several Mathematics A-Level classes, and a college atmosphere where more mature demands are often made of students.

and schools

Within the school sector, there still exist some **grammar schools**, selective at 11 or 12, in England and Wales, complemented by **technical or secondary schools**, but the vast majority of 11–18 students in the state sector are educated in 11–16 or 11–18 **comprehensives** (there is some variety in age ranges across the country). City Technology Colleges and Specialist Schools of various sorts are nominally comprehensive (although better resourced than the average), although they may select some part of their intake by testing for ‘aptitude’. 11–16 schools frequently have difficulty recruiting mathematics teachers with the skills and knowledge necessary to stretch students to give the best foundations for post-16 work, and even in 11–18 schools the very thin spread of mathematical expertise in the teaching profession, particularly in more expensive areas, can lead to many students’ abilities not being as well-developed as one would wish. There is at present some pressure from the LSC for the post-16 provision of several such institutions to amalgamate, in part to give students wider choice, and at least in part to try to tackle such shortages. Students entering post-16 institutions from comprehensive schools are likely to have been set for mathematics, but even so, will probably learnt the subject in groups with a wide ability range: a top set might typically have contained the top 25% of the ability range, with the result that students emerge tolerant of one another’s abilities, but having had less targeted tuition than in selective institutions.

Some 7% of the secondary population attend **independent schools**, the best of which provide unrivalled teaching and learning resources and opportunities, as well as expertise, but there is enormous variation in quality across the sector.

Scotland, Wales and Northern Ireland

In **Scotland**, students take Standard Grade examinations, typically in eight subjects, at age 16, followed by Higher (comparable to AS) at 17 (matriculation), and Advanced Higher (comparable to A2) at 18. 11% of the age cohort study mathematics to this level, with 2% of these 11% gaining the top grade. Post-16 courses involve in-course unit tests, but qualifications are graded on performance in end-of-course examinations. There is just one Scottish examining board, SQA. Secondary teachers are required to be well-qualified in their teaching subject, and the supply of mathematics teachers is not a major issue.

In **Wales** the variety of available courses post-16 and the range of institutions are similar to that in England. Most students sit the examinations of the WJEC.

In **Northern Ireland,** most areas still operate on a grammar school system, with a strong supply of mathematics teachers. About 12% of the age cohort take GCSE Additional Mathematics, covering much of the P1, M1 and S1 specifications – an excellent preparation for the A Level course – and take-up of mechanics continues to be higher than that of statistics, in contrast to the trend in England and Wales. Most students sit the examinations of the CCEA.

How will they have studied?

The variety of institutions from which a student may arrive at university, is vast, and the student's experience will vary even within the same type of teaching and learning community. Some students will be used to having work set every lesson, and marked frequently and in detail; others will have been much more self-sufficient and independent. Some will have been fully stretched and worked largely with their intellectual peers; for others the background is one of very mixed ability and this may be the first time they have worked with other students of comparable ability. Some have fitted mathematical study round a full-time job or family responsibilities; others have been able to devote as much time as they wanted to study. Many will be mixing with those of very different social backgrounds for the first time.

What will they know from sixth-form mathematics?

All students with A Level Mathematics, whether taking examinations before or after the revisions for teaching from September 2004, will have met a core of pure mathematics. In addition, they will have taken some applied mathematics modules (mechanics and/or statistics and/or discrete mathematics), but two students will not necessarily have any applied exposure in common. Both AS and A level Mathematics specifications require an emphasis on correct use and understanding of mathematical terminology and argument, and some basic ideas of mathematical modelling.

what will have been taught

core content

In the A-Level core, students will have met basic ideas of algebra and functions: manipulation of indices and surds; solutions of linear and quadratic equations and inequalities and some simultaneous equations; partial fractions and the remainder theorem; functions and transformations including the modulus, exponential and logarithmic functions and basic manipulation of these. They will have encountered coordinate geometry of straight lines and circles; cartesian and parametric equations of curves; basic ideas of sequences, including definition by recurrence relation; arithmetic, geometric and binomial series; radian measure; the six trigonometric functions including the use of various formulae and solution of simple trigonometric equations; students will have met differentiation of polynomials, exponential and natural logarithmic functions and of sin, cos and tan; differentiation of products and quotients, chain rule and implicit differentiation; geometric applications of differentiation and formation of simple differential equations; integration of powers of x, including 1/x, and of sin x and cos x; the use of substitution and integration by parts; evaluation of areas under curves and volumes of revolution; solution of simple first order differential equation with variables separable only. Students will have located approximate roots of continuous $f(x) = 0$ by change of sign, and of $f(x) = x$ by simple iterative methods; used trapezium rule for numerical integration; met vectors in 2 and 3 dimensions including addition and multiplication by scalars, position vectors, distance between 2 points, vector equations of lines, and scalar product and have encountered geometrical interpretation of these.

what will have been learnt

These are the ideas that students will have met, not that they will necessarily have mastered. When they took the last examinations, even A-grade students were unlikely to have mastered all of the above, particularly the harder calculus indicated, and any material which was not understood in depth at that stage is unlikely to be in good working order by late September or early October when they arrive at university. C-grade candidates will typically show relative weakness in algebraic manipulation and calculus, and E-grade students are likely to have very weak algebra and poor understanding of much of the calculus they have encountered. Students may well have experienced being taught in very mixed ability groups at school. If they are on the right course, they will often find themselves in a much more homogeneous grouping at university, using their mathematics in an area of particular interest. They may well find this stimulating mathematically. Nevertheless, the university will be able to assume mastery of very few specific items of content.

How do students feel about the transition to university mathematics?

before

- I enjoy maths at school: I suppose I expect it to be a bit like that, only harder. I'm looking forward to some of the new bits of maths that I haven't met before. (Year 13, hoping to read Mathematics at Cambridge)
- I want to do maths because I like the puzzling side: you know, getting stuck and then somehow finding out how to do it. I want to do more of it, and some of the Pure Maths you read about in prospectuses sounds really interesting. It'll be weird having much less direct and personal teaching though. I hope I can get to know other people who enjoy working at maths in groups – I really enjoy that. (Year 12, hoping to teach Mathematics eventually, expected to get a C or B)
- It will be lots of hard work, with a faster pace and a bigger workload than A-level. (Year 12, hoping to read Mathematics at Oxford)
- Will the maths I do at university be a lot harder than this? I can manage this but only just. I'm hoping that it'll be more like what we do in mechanics, because that isn't as abstract as some of what we do in Pure. Who do I ask if I get really stuck? (Year 13, hoping to do Engineering at Southampton, expected to get a B)

after

- My background put me head and shoulders above the rest of the class. I struggled to keep up with the others in the Further Maths group at school, but that gave me the experience of persevering, and the confidence that if I worked at it, I'd understand eventually, as well as the extra skills. (Aeronautical Engineering at Loughborough, A in Mathematics and B in Further Mathematics)
- This was right for me: I'm getting a really confident grasp of the ideas now, and I realise I wouldn't have been able to build on them if I'd gone straight to university (student on HND in Mechanical Engineering, D in Mathematics, U in Physics, rejected by his preferred universities)
- The maths is fine, I could keep up with anyone else on the course, we repeated a lot of what we'd done at school, but living out in the first year, with people I didn't know, just made it all too much of a change – I was thoroughly miserable. (Mathematics at Oxford Brookes, DEE at A Level)
- It's been a steep learning curve: at school, I could usually just about figure things out, even STEP questions if we persevered enough, but now... (Mathematics at Cambridge)
- The course is not at all what I expected: there were lots of technical words I didn't understand of course, but those I thought I understood, turned out to mean something completely different at university (Mathematics and Physics, Keele)
- It's great: I really struggled to keep up at school, but now it all makes sense. It's in a context, and I'm as able as the next student – I don't feel I'm holding people back (Engineering at UWE, DEE at A Level)
- I just couldn't keep it up. The lectures were great, and the other students really interesting, but I'd be worrying about things at home, and there were so many competing pressures (mature student, Reading)

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